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DESIGN METHOD FOR TWO-DIMENSIONAL CHANNELS FOR COMPRESSIBLE FLOW WITH APPLICATION TO HIGH-SOLIDITY CASCADES

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DESIGN METHOD FOR TWO-DIMENSIONAL CHANNELS FOR COMPRESSIBLE FLOW WITH APPLICATION TO HIGH-SOLIDITY CASCADES

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SUMMARY

A procedure is presented for the design of two-dimensional channels for compressible nonviscous flow. The method requires boundary conditions consisting of the shape of one channel surface and the velocity distribution on that surface. The process consists of the step-by-step computation of an arbitrary number of streamlines within the channel. Two variations of the method are presented: One variation is based on an assumed constant streamline curvature along an equipotential line, and the other is based on an assumed vortex-type variation. Tables for the use of both methods are presented for a range of Mach numbers and values of the ratio of specific heats.

INTRODUCTION

In order to obtain satisfactory performance from gas turbines, a blade-profile design yielding prescribed velocity distributions is desirable. Proposed procedures for blade-profile design from specified surface velocities known as inverse procedures, such as those given in references 1 and 2, utilize conformal transformation or interference techniques. Such methods are limited in application by the assumption of incompressible flow and by the lengthy computations required.

For closely spaced blades of high solidity, such as those encountered in axial-flow turbines, the application of channel methods has been found useful. Stodola (reference 3) presents Flugel's stream-filament technique for determining the velocities in a known channel by graphical integration. A simplified method for such analysis is presented in reference 4.

A procedure based on stream-filament techniques for the design of two-dimensional channels for compressible nonviscous flow has been developed at the NACA Lewis laboratory and is

presented herein. The method requires boundary conditions consisting of the shape of one channel surface and the velocity distribution on that surface. Although the method is not a completely inverse solution, it is useful in the design of blades for high-solidity cascades. The proposed procedure deals only with the channel between adjacent blades and is inapplicable to the regions in the vicinity of the nose and the tail. A similar technique for incompressible flow is given by von Mises (reference 5). The specification of the shape and the velocity distribution on one channel surface constitutes an over-determination of desired conditions in that the specification of only a short continuous length of streamline and the velocities along it are needed to determine the flow in the complete field. The method of solution, however, is based on satisfying the conditions of continuity and irrotational flow at a series of points along the specified streamline and the fairing of a curve to represent the solution. The solution is thus an approximate one and the velocities in the resulting channel should be computed to determine the degree of satisfaction of the specified conditions.

Further variation from specified conditions will result from the fact that the inlet flow required to satisfy theoretically the specified conditions in the channel is unknown. This variation will undoubtedly be small as experiments of high-solidity cascades have shown that the flow in the channel sections of cascades is relatively insensitive to the inlet conditions.

In the application to cascade design of the method presented herein, specification of the boundary conditions as those for the suction surface of the blade is desirable. An arbitrary number of stream filaments is successively constructed starting from the assigned surface, each carrying a specified fraction of the total weight flow. The streamline bounding the last stream filament is taken as the channel boundary and as the pressure surface of an adjacent blade. The orthogonal spacing between a given streamline and the next streamline constructed from it is approximated by a spacing taken perpendicular to the given streamline. This spacing is computed by integration of the flow equations with an assumed variation of streamline curvature along an equipotential line. Two variations are considered herein; (1) a constant curvature, and (2) a vortex-type variation.

Weinig (reference 6) presents a solution to a similar problem by the graphical construction of an orthogonal flow net in which the spacings between streamlines, for equal potential increments

along the assigned streamline, are taken as equal to the potential spacings with corrections for compressibility. The streamlines are constructed for several potential increments and an extrapolation yields the final flow net.

THEORY OF METHOD

The problem may be stated as follows: Given the shape of one of the channel boundaries, the desired velocity distribution thereon, and the weight flow to be carried by the channel, find the opposite boundary. In the case of a cascade, the shape of the suction surface of a blade and the velocity distribution on that surface will be assumed and the pressure surface of the adjacent blade will be sought. The form in which the initial surface and the velocity distribution are assumed and the construction of an adjacent streamline are shown in figure 1. The orthogonal spacing between a given streamline and the next streamline constructed from it is approximated by a spacing l taken perpendicular to the given streamline.

Basic equations. - The continuity equation for two-dimensional flow in a channel may be written as

$$w = \int \gamma V \, dL \tag{1}$$

where

w channel weight flow per unit height

γ weight density

V velocity

dL line element perpendicular to direction of flow

All symbols are redefined in appendix A.

The condition for irrotational flow is expressed as

$$\frac{dV}{dL} = -CV \tag{2}$$

2. The curvature C is assumed to vary as in vortex flow such that

$$\frac{X_{i+1}}{C_{i+1}} = \frac{X_i}{C_i}$$

The application of the two methods resulting from these assumptions will be discussed in a subsequent section.

In order to minimize the inaccuracies due to these assumptions, they are applied to only a part of the total channel carrying the specified fraction n of the total weight flow. Equation (4) can then be solved for the unknown velocity parameter X_{i+1} and the approximate l computed from the irrotation-flow equation. With this value known, the streamline ψ_{i+1} can be graphically constructed and its curvature, in turn, used for the construction of the next streamline.

Solution for assumption of constant curvature. - If C is assumed equal to C_i, equation (4) becomes

$$-C_i \, nf(w) = \int_{X_i}^{X_{i+1}} (1 - X^2)^{\frac{1}{k-1}} dX \tag{5}$$

The integral can be expressed as

$$-C_i \, nf(w) = \int_0^{X_{i+1}} (1 - X^2)^{\frac{1}{k-1}} dX - \int_0^{X_i} (1 - X^2)^{\frac{1}{k-1}} dX \tag{6}$$

where the unknown is the value X_{i+1} representing the velocity on the streamline forming the boundary of the stream filament carrying the weight flow specified by $nf(w)$.

For a known value of k, the integral

$$g(X_r) = \int_0^{X_r} (1 - X^2)^{\frac{1}{k-1}} dX$$

can be computed as a function of the limit X_r. This computation has been carried out in the manner illustrated in appendix B and the results tabulated in tables I and II.

Equation (6) can be written as

$$-C_i nf(w) = g(X_{i+1}) - g(X_i) \tag{7}$$

With the known curvature C_i, the assigned fraction of the weight flow $nf(w)$, the known velocity parameter X_i, and $g(X_i)$ found from the tabulated values, equation (7) is solved for $g(X_{i+1})$ and X_{i+1} found from table I.

If X_{i+1} is known, the approximate spacing l_c (subscript c denoting constant-curvature assumption) can be found for the assumption of constant curvature by integrating equation (2).

$$\log_e \frac{V_{i+1}}{V_i} = -C_i \, l_c = \log_e \frac{X_{i+1}}{X_i}$$

or

$$l_c = -\frac{1}{C_i} \log_e \frac{X_{i+1}}{X_i} \tag{8}$$

The value l_c is computed at a point on the known streamline and graphically laid out normal to the streamline at that point.

By computing l for a number of points, the adjacent streamline ψ_{i+1} can be drawn. (See fig. 1.) With the curvature of this new streamline C_{i+1}, which can be readily computed, the initial curvature C_i, and the spacing l, a closer approximation to the velocity on the new streamline X_{i+1} can be found from equation (8) using an average curvature.

$$X_{i+1} = X_i \, e^{-\frac{\left(C_i + C_{i+1}\right) l}{2}} \tag{9}$$

Further refinement of l is not required unless the average curvature is greatly different from the initial value C_i. If the velocity parameter X_{i+1} is known, the next streamline ψ_{i+2} can be constructed. The process is repeated until channel space has been computed to allow for all the specified weight flow.

Solution for assumption of vortex variation. - A vortex variation of the form

$$C = \frac{C_i}{V_i} V = \frac{C_i}{X_i} X \tag{10}$$

is inserted into equation (4).

$$nf(w) = -\int_{X_i}^{X_{i+1}} \frac{X_i}{C_i} \frac{1}{X} \left(1 - X^2\right)^{\frac{1}{k-1}} dX$$

If the constant X_i/C_i is removed from the integrand and the integral is replaced by the sum of two integrals,

$$-\frac{C_i}{X_i} nf(w) = \int_0^{X_{i+1}} \frac{1}{X} \left(1 - X^2\right)^{\frac{1}{k-1}} dX - \int_0^{X_i} \frac{1}{X} \left(1 - X^2\right)^{\frac{1}{k-1}} dX \tag{11}$$

At the lower limits the integrals are improper. As only the differences between the two integrals are required, a lower limit of 0.01 will therefore be used. By definition,

$$h(X_r) = \int_{0.01}^{X_r} \frac{1}{X} \left(1 - X^2\right)^{\frac{1}{k-1}} dX$$

For a known value of k, the integral $h(X_r)$ can be computed as a function of the limit X_r. This computation has been performed in the manner illustrated in appendix C and the results tabulated in tables III and IV.

Equation (11) can then be written

$$-\frac{C_i}{X_i}\,nf(w) = h(X_{i+1}) - h(X_i) \tag{12}$$

and the value of X_{i+1} may be found.

The channel spacing can then be computed by integrating equation (2) with the curvature variation of equation (10)

$$\frac{dV}{V^2} = -\frac{C_i}{V_i}\,dL$$

or

$$\frac{dX}{X^2} = -\frac{C_i}{X_i}\,dL$$

By integration between X_i and X_{i+1}

$$l_v = \frac{1}{C_i}\left(\frac{X_i}{X_{i+1}} - 1\right) \tag{13}$$

where the subscript v indicates vortex-type-variation assumption.

With this value for l_v, the new streamline can be constructed and a closer approximation to the velocity parameter X_{i+1} can be obtained by use of equation (9).

Comparison of curvature assumptions. - For the same initial condition, the effect of the curvature assumption on the computed value of l is a function of the step size so that for a specific application the effect is dependent on the number of steps $1/n$ used in the computation. Without a knowledge of the true variation

in the streamline curvature, the absolute effect of the assumption cannot be determined. From several typical design problems, the vortex-type variation was found to more closely approximate the variations obtained.

For the same initial conditions of C_1 and X_1 and for the same value of $nf(w)$, the differences in l for the constant curvature and vortex-type-variation assumptions can be computed. These differences are plotted in figure 2 as a function of step-size parameter $C_1 l_v$ with X_1 as a parameter. It can be seen that the constant-curvature assumption yields a higher value of l than does the vortex-type variation. For most design problems, the maximum value of $C_1 l_v$ (or $C_1 l_c$) can be kept below 0.3 without an excessive number of steps. For this value, the differences in l are usually less than the accuracy of the graphical construction process.

APPLICATION OF METHOD

Selection of prescribed velocity and boundary shape. - In blade-design work, the suction surface is usually the more critical surface. As the proposed method allows any streamline to be specified, it appears to be of greater advantage to fix the suction-surface conditions. Any arbitrary surface and velocity distribution may be assigned, but they might not result in a physically possible blade. As a basis for assigning the initial surface and velocity, it has been found convenient to analyze first an arbitrary channel drawn to give the appearance of a conventional blade. The velocities in this channel can be easily found by the direct stream-filament method of reference 4. The suction-surface shape or the velocity distribution or both can then be adjusted and utilized as the initial values in the proposed method. (Such adjustments should be carefully made to prevent excessive thickness or thinness of the resulting blade. For subsonic flow, an increase in the assigned velocity will result in a narrower channel, thus a thicker blade, and vice versa. An increase in curvature on the suction surface will result in a wider channel at that point.)

Boundary-layer theory has shown that flow separation is most likely to occur in a region where deceleration takes place. In order to minimize the danger of separation, information on boundary-layer flow should be employed to limit the rate of flow deceleration along the suction surface. In the absence of any reliable information on boundary layers, the velocity over as large a portion of the suction surface as possible should be either constant or increasing.

Determination of weight-flow parameter. - The weight-flow parameter $f(w)$ may be readily computed from the known velocity diagrams and the blade spacing by

$$f(w) = sX_\infty\left(1 - X_\infty^2\right)^{\frac{1}{k-1}} \cos\theta \qquad (14)$$

(See appendix D for development.)

where

s blade pitch

X_∞ relative inlet-velocity parameter at infinity

θ angle between relative velocity and its axial component at infinity

The parameter $f(w)$ is related to the mass-flow parameter μ of reference 4 by

$$\mu = \frac{f(w)}{l_w}$$

where l_w is the total channel width measured along an orthogonal line.

For design problems where the flow per unit blade height is not constant through the channel, the weight-flow parameter can be varied to conform to the expected distribution in weight flow per unit blade height.

Selection of number of steps. - The number of steps to be used in the computation can be set from considerations of the desired accuracy. As the errors involved in a curvature assumption increase with step size, it is apparent that small steps are desirable; as the graphical construction of each streamline introduces errors, however, the least number of steps consistent with theoretical accuracy should be used.

A large number of small steps would, theoretically, yield the highest accuracy. Two factors contributed errors to the computation; namely, (1) the assumption on the curvature variation and (2) the use of a straight line in place of a true orthogonal for constructing l. Without knowing the true curvature variation, complete evaluation of the effect of these assumptions is impossible.

With low initial velocities and low curvatures, relatively larger steps can be used than for high velocities and curvatures. For most turbine blades, five steps $(n = 1/5)$ should be adequate; whereas three steps can be used for sections not requiring accurately specified velocities.

Selection of curvature variation. - The comparison of l as computed for the vortex variation and for constant curvature is shown in figure 2. For low initial velocities and curvatures, the difference is very small; unless a high degree of accuracy is desired and large steps used, the differences are small compared with the errors involved in graphically constructing l and computing curvatures. From illustrative examples, the vortex variation was found to more closely approximate the measured variation than did the constant-curvature assumption.

SUGGESTED PROCEDURE FOR COMPUTATION

The following step-by-step procedure is suggested:

1. Assign suction-surface shape ψ_0 and velocity distribution X_0. The curvature of the assigned streamline C_0 can be found by graphical or computational methods.

2. Compute the weight-flow function $f(w)$ and set n for the desired number of steps. For a departure from two-dimensional flow where the value of $f(w)$ is not constant, the values at the inlet and the outlet of the channel should be computed and a variation between these values assumed.

3. For a number of points (10 to 15) on the given streamline, compute l.

(a) For vortex variation. - For the initial velocity parameter X_0, obtain the value of $h(X_0)$ from table III. By use of equation (12) compute $h(X_1)$ and obtain X_1 from table III. Compute l_v by equation (13).

(b) For constant curvature. - For the initial velocity parameter X_0, obtain the value of $g(X_0)$ from table I. By use of equation (7) compute $g(X_1)$ and obtain X_1 from table I. Compute l_c by equation (8).

4. With the values of l known, construct the streamline ψ_1 as shown in figure 1 and compute its curvature C_1.

5. Compute the velocity on the new streamline by equation (9). The curvature and velocity values in this equation are those at the points of intersection of the l line with the respective streamlines.

6. Repeat steps 3 and 4 for the assigned number of stream filaments and take the last streamline $\psi_{1/n}$ as the channel boundary.

7. Compute the velocities in the channel to determine the degree to which the specified surface velocities have been satisfied. The method of reference 4 can be used to good advantage although it may be necessary to carry the method beyond the usual first approximation of consideration of the channel in a single step.

EFFECT OF VARIATIONS IN PRIMARY PARAMETERS

As an aid in adjusting assigned velocity and curvature values as well as an indication of the needed accuracy in curvature determination, a study was made of the effect of variations in the primary parameters C_i and X_i on l_v.

The effect of a variation in initial curvature on the computation of l_v was found by taking the partial derivatives of equations (12) and (13) with respect to C_m. As

$$\frac{\partial l}{\partial C_i} \approx \frac{\Delta l}{\Delta C_i}$$

and by definition

$$\frac{El}{EC_i} = \frac{\dfrac{\Delta l}{l}}{\dfrac{\Delta C_i}{C_i}}$$

it can be shown that

$$\frac{El_v}{EC_1} \approx \frac{X_1\left[h(X_1) - h\left(\frac{X_1}{1 + C_1 l_v}\right)\right]}{C_1 l_v\left(\frac{X_1}{1 + C_1 l_v}\right)\left[1 - \left(\frac{X_1}{1 + C_1 l_v}\right)^2\right]^{\frac{1}{k-1}}} - 1 \qquad (15)$$

Values of this ratio El_v/EC_1 have been computed for values of $C_1 l_v$ from 0 to 1.0 and X_1 of 0.2, 0.3, and 0.4 for k of 1.4. The results are shown in figure 3.

Similarly, the effect of a variation in X_1 on the computation of l_v was found to be

$$\frac{El_v}{EX_1} = \frac{\frac{\Delta l_v}{l_v}}{\frac{\Delta X_1}{X_1}}$$

$$\approx \frac{1 + C_1 l_v}{C_1\ l_v}\left\{1 - \left[\frac{1 - X_1^{\ 2}}{1 - \left(\frac{X_1}{C_1\ l_v + 1}\right)^2}\right]^{\frac{1}{k-1}} - \frac{h(X_1) - h\left(\frac{X_1}{1 + C_1\ l_v}\right)}{\left[1 - \left(\frac{X_1}{1 + C_1\ l_v}\right)^2\right]^{\frac{1}{k-1}}}\right\} \qquad (16)$$

Values of this ratio El_v/EX_1 have been computed for values of $C_1 l_v$ from 0 to 1.0 and X_1 of 0.2, 0.3, and 0.4, for k of 1.4. The results are shown in figure 4.

Illustrative Example

The channel portion of a typical blade was designed by the methods presented herein. Assigned values were

$$X_\infty = 0.2062 \left(\frac{V_\infty}{V_{cr}} = 0.505\right)$$

$$\frac{c_x}{s} = 1.475$$

$$\theta = 41.3^\circ$$

where

c_x axial width

An arbitrary suction-surface velocity and suction-surface shape were assumed as X_0 and ψ_0, as shown in figure 5. For $n = 1/3$, the channel was constructed using both the vortex-type variation and the constant-curvature method. The design plot for the constant-curvature method is shown in figure 5. The third streamline ψ_3 is taken as the pressure surface of the adjacent blade. A comparison of the channels found by the two methods is shown in figure 6. In the channel section corresponding to the highly curved portion of the suction surface, the channel width was greater for the constant-curvature computation. This difference is in accordance with the theoretical differences previously noted. Beyond the 40-percent chord position, the two channels are in close agreement. The differences in pressure-surface profile, which show a wider channel for the vortex variation for a small section of the blade surface, are possibly due to errors in graphical construction and in curvature computation.

Lewis Flight Peopulsion Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX A

SYMBOLS

The following symbols are used in this report:

C	curvature (reciprocal of radius of curvature)
c_p	specific heat at constant pressure
c_x	axial width
E	prefix to denote fractional variation
$f(w)$	weight-flow parameter, $\dfrac{w}{\gamma' \sqrt{2c_pT'}}$
$g(X_r)$	integral function, $\displaystyle\int_0^{X_r} \left(1 - X^2\right)^{\frac{1}{k-1}} dX$
$h(X_r)$	integral function, $\displaystyle\int_{0.01}^{X_r} \frac{1}{X}\left(1 - X^2\right)^{\frac{1}{k-1}} dX$
k	ratio of specific heats, c_p/c_v
L	length along velocity potential line
l	approximate width of stream filament
n	specified fraction of weight flow
s	blade pitch
T	absolute temperature
V	velocity
w	channel weight flow per unit height
X	dimensionless velocity parameter, $\dfrac{V}{\sqrt{2c_pT'}}$

γ weight density

Δ prefix to indicate change

θ angle between relative inlet velocity and its axial component at infinity

ψ designation of streamlines

Subscripts:

0 assigned conditions

1,2,3 index of streamlines

c computed for constant curvature

cr conditions at critical sonic velocity

i general index

r limit value

v computed for vortex-type variation

w total width

x axial

Superscripts:

' stagnation state

APPENDIX B

COMPUTATION OF $g(X_r)$

The integral

$$g(X_r) = \int_0^{X_r} (1 - X^2)^{\frac{1}{k-1}} dX$$

can be readily integrated by expanding the integrand in a power series and integrating termwise.

$$\left(1 - X^2\right)^{\frac{1}{k-1}} = 1 - \frac{1}{k-1} X^2 + \frac{2-k}{2(k-1)^2} X^4 - \frac{(2-k)(3-2k)}{6(k-1)^3} X^6 + \ldots$$

$$g(X_r) = \int_0^{X_r} (1 - X^2)^{\frac{1}{k-1}} dX = X_r - \frac{X_r^3}{3(k-1)} + \frac{(2-k)X_r^5}{10(k-1)^2} - \frac{(2-k)(3-2k)X_r^7}{42(k-1)^3} + \ldots$$

For a Mach number of 1.0 and a k of 1.40, the value of X is 0.40825 and for lower values of k, the value of X decreases. For the range of X from 0 to 0.41 and for k from 1.28 to 1.40, the first four terms shown in the previous expansion were sufficient to compute $g(X_r)$ accurately to five decimal places.

The values of the integral $g(X_r)$ are given in table I for values of the argument X_r from 0 to X_{cr} in increments of 0.005 for k of 1.40, 1.38, 1.36, 1.34, 1.32, 1.30, and 1.28.

Values of $g(X_r)$ for critical (sonic) velocities for the values of k used are tabulated in table II. For critical flow

$$X_{cr} = \sqrt{\frac{k-1}{k+1}}$$

APPENDIX C

COMPUTATION OF $h(X_r)$

The integral

$$h(X_r) = \int_{0.01}^{X_r} \frac{1}{X} (1 - X^2)^{\frac{1}{k-1}} dX$$

can be readily integrated by expanding the integrand in a power series and integrating termwise.

$$\frac{1}{X} (1 - X^2)^{\frac{1}{k-1}} dX = \left[\frac{1}{X} - \frac{1}{k-1} X + \frac{2-k}{2(k-1)^2} X^3 - \frac{(2-k)(3-2k)}{6(k-1)^3} X^5 + \ldots\right] dX$$

When this equation is integrated, it yields

$$h(X_r) = \log_e X - \frac{X^2}{2(k-1)} + \frac{(2-k)}{8(k-1)^2} X^4 - \frac{(2-k)(3-2k)}{36(k-1)^3} X^6 + \ldots\Bigg]_{0.01}^{X_r}$$

Values of this integral for X_r from 0.010 to X_{cr} have been computed and are given in table III for $k = 1.40$, 1.38, 1.36, 1.34, 1.32, 1.30, and 1.28.

Values of $h(X_r)$ for critical velocities are given in table IV.

APPENDIX D

DETERMINATION OF WEIGHT-FLOW PARAMETER f(w)

The weight-flow parameter may be readily determined from the velocity diagrams by application of the continuity equation to conditions at infinity relative to the blade. The channel weight flow per unit blade height may be written as

$$w = \gamma s V_{x,\infty}$$

The term f(w) is defined as

$$f(w) = \frac{w}{\gamma' \sqrt{2c_p T'}}$$

so that

$$f(w) = \frac{\gamma}{\gamma'} \frac{sV_{x,\infty}}{\sqrt{2c_p T'}}$$

Inasmuch as

$$\frac{\gamma}{\gamma'} = \left[1 - \frac{k-1}{k+1}\left(\frac{V}{V_{cr}}\right)^2\right]^{\frac{1}{k-1}}$$

and

$$V_{x,\infty} = V_\infty \cos\theta$$

and

$$X = \sqrt{\frac{k-1}{k+1}} \frac{V}{V_{cr}}$$

then

$$f(w) = sX_\infty \left(1 - X_\infty^2\right)^{\frac{1}{k-1}} \cos\theta$$

For rotor blades $f(w)$ must be computed relative to the rotating blades.

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TABLE I - VALUES OF FUNCTION $g(X_r) = \int_0^{X_r} (1-X^2)^{\frac{1}{k-1}} dX$

Velocity parameter X_r	Ratio of specific heats of gas, k						
	1.40	1.38	1.36	1.34	1.32	1.30	1.28
0	0	0	0	0	0	0	0
.005	.00500	.00500	.00500	.00500	.00500	.00500	.00500
.010	.01000	.01000	.01000	.01000	.01000	.01000	.01000
.015	.01500	.01500	.01500	.01500	.01500	.01500	.01500
.020	.01999	.01999	.01999	.01999	.01999	.01999	.01999
.025	.02499	.02499	.02499	.02498	.02498	.02498	.02498
.030	.02998	.02998	.02998	.02997	.02997	.02997	.02997
.035	.03496	.03496	.03496	.03496	.03496	.03495	.03495
.040	.03995	.03994	.03994	.03994	.03993	.03993	.03993
.045	.04492	.04492	.04492	.04491	.04490	.04490	.04489
.050	.04990	.04989	.04988	.04988	.04987	.04986	.04985
.055	.05486	.05485	.05485	.05484	.05483	.05482	.05480
.060	.05982	.05981	.05980	.05979	.05978	.05976	.05974
.065	.06477	.06476	.06475	.06473	.06472	.06470	.06467
.070	.06972	.06970	.06968	.06966	.06964	.06962	.06960
.075	.07465	.07463	.07461	.07459	.07456	.07453	.07450
.080	.07958	.07955	.07953	.07950	.07947	.07943	.07939
.085	.08449	.08446	.08443	.08440	.08436	.08432	.08427
.090	.08940	.08936	.08933	.08929	.08924	.08920	.08914
.095	.09429	.09425	.09421	.09416	.09411	.09405	.09399
.100	.09917	.09913	.09908	.09902	.09896	.09890	.09882
.105	.10404	.10399	.10393	.10387	.10380	.10372	.10363
.110	.10890	.10884	.10878	.10870	.10862	.10853	.10843
.115	.11374	.11367	.11360	.11352	.11343	.11333	.11321
.120	.11857	.11850	.11841	.11832	.11822	.11810	.11797
.125	.12338	.12330	.12321	.12310	.12299	.12285	.12270
.130	.12818	.12809	.12798	.12787	.12774	.12759	.12742
.135	.13297	.13286	.13274	.13261	.13247	.13230	.13211
.140	.13773	.13762	.13749	.13734	.13718	.13699	.13678
.145	.14248	.14235	.14221	.14205	.14187	.14166	.14143
.150	.14722	.14707	.14691	.14673	.14653	.14631	.14605

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TABLE I - VALUES OF FUNCTION $g(X_r) = \int_0^{X_r} (1-X^2)^{\frac{1}{k-1}} dX$ - Continued.

Velocity parameter X_r	Ratio of specific heats of gas, k						
	1.40	1.38	1.36	1.34	1.32	1.30	1.28
0.150	0.14722	0.14707	0.14691	0.14673	0.14653	0.14631	0.14605
.155	.15193	.15177	.15160	.15140	.15118	.15093	.15065
.160	.15663	.15645	.15626	.15604	.15580	.15553	.15522
.165	.16130	.16111	.16090	.16066	.16040	.16010	.15976
.170	.16596	.16575	.16552	.16526	.16498	.16465	.16428
.175	.17060	.17037	.17012	.16984	.16952	.16917	.16877
.180	.17521	.17496	.17469	.17439	.17405	.17366	.17323
.185	.17981	.17954	.17924	.17892	.17855	.17813	.17766
.190	.18438	.18409	.18377	.18342	.18302	.18257	.18206
.195	.18893	.18862	.18827	.18789	.18746	.18698	.18643
.200	.19345	.19312	.19275	.19234	.19188	.19136	.19077
.205	.19796	.19760	.19718	.19676	.19626	.19571	.19507
.210	.20244	.20205	.20162	.20115	.20062	.20002	.19934
.215	.20689	.20648	.20602	.20552	.20495	.20431	.20358
.220	.21132	.21088	.21039	.20985	.20925	.20856	.20779
.225	.21572	.21525	.21474	.21416	.21351	.21278	.21196
.230	.22010	.21960	.21905	.21844	.21775	.21697	.21610
.235	.22445	.22392	.22333	.22268	.22195	.22113	.22020
.240	.22878	.22821	.22759	.22690	.22612	.22525	.22426
.245	.23307	.23248	.23181	.23108	.23026	.22933	.22828
.250	.23734	.23671	.23601	.23523	.23436	.23338	.23228
.255	.24158	.24091	.24017	.23935	.23843	.23740	.23623
.260	.24580	.24509	.24430	.24344	.24247	.24138	.24014
.265	.24998	.24922	.24841	.24749	.24647	.24532	.24401
.270	.25413	.25334	.25247	.25151	.25043	.24922	.24785
.275	.25825	.25742	.25651	.25549	.25436	.25309	.25164
.280	.26235	.26147	.26051	.25944	.25825	.25691	.25540
.285	.26641	.26549	.26448	.26336	.26211	.26070	.25912
.290	.27044	.26948	.26842	.26724	.26593	.26445	.26279
.295	.27444	.27343	.27232	.27108	.26971	.26816	.26642
.300	.27840	.27734	.27618	.27489	.27345	.27184	.27001

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TABLE I.- VALUES OF FUNCTION $g(X_r) = \int_0^{X_r} (1-X^2)^{\frac{1}{k-1}} dX$ - Concluded.

Velocity parameter X_r	Ratio of specific heats of gas, k						
	1.40	1.38	1.36	1.34	1.32	1.30	1.28
0.300	0.27840	0.27734	0.27618	0.27489	0.27345	0.27184	0.27001
.305	.28234	.28123	.28001	.27866	.27715	.27547	.27356
.310	.28624	.28508	.28380	.28239	.28082	.27906	.27707
.315	.29010	.28889	.28756	.28609	.28445	.28261	.28053
.320	.29394	.29268	.29128	.28975	.28803	.28612	.28395
.325	.29774	.29642	.29497	.29337	.29158	.28958	.28733
.330	.30150	.30013	.29862	.29695	.29509	.29301	.29066
.335	.30523	.30380	.30223	.30049	.29856	.29639	.29396
.340	.30893	.30744	.30580	.30399	.30198	.29973	.29720
.345	.31259	.31104	.30934	.30746	.30537	.30303	.30040
.350	.31621	.31460	.31284	.31088	.30871	.30629	.30356
.355	.31980	.31813	.31629	.31427	.31202	.30950	.30667
.360	.32335	.32162	.31971	.31761	.31528	.31267	
.365	.32687	.32507	.32310	.32092	.31850	.31580	
.370	.33035	.32848	.32644	.32418	.32167		
.375	.33379	.33186	.32974	.32740	.32481		
.380	.33719	.33520	.33300	.33058			
.385	.34056	.33849	.33622	.33372			
.390	.34389	.34175	.33940				
.395	.34718	.34497	.34255				
.400	.35043	.34815					
.405	.35365						
.410	.35682						

TABLE II.- VALUES OF $g(X_r)$ FOR CRITICAL VELOCITIES

k	X_{cr}	$g(X_{cr})$
1.40	0.40825	0.35571
1.38	.39957	.34788
1.36	.39056	.33976
1.34	.38118	.33132
1.32	.37139	.32254
1.30	.36115	.31339
1.28	.35044	.30384

TABLE III - VALUES OF FUNCTION $h(X_r) = \int_{0.01}^{X_r} \frac{1}{X}(1-X^2)^{\frac{1}{k-1}}\, dX$

Velocity parameter X_r	Ratio of specific heats of gas, k						
	1.40	1.38	1.36	1.34	1.32	1.30	1.28
0.010	0	0	0	0	0	0	0
.015	.40531	.40530	.40529	.40528	.40527	.40526	.40524
.020	.69277	.69275	.69273	.69271	.69268	.69265	.69261
.025	.91564	.91600	.91556	.91552	.91547	.91542	.91535
.030	1.09761	1.09756	1.09750	1.09744	1.09736	1.09728	1.09718
.035	1.25136	1.25128	1.25120	1.25111	1.25101	1.25089	1.25076
.040	1.38442	1.38432	1.38421	1.38409	1.38395	1.38380	1.38362
.045	1.50167	1.50155	1.50141	1.50125	1.50107	1.50087	1.50064
.050	1.60644	1.60628	1.60611	1.60591	1.60569	1.60544	1.60516
.055	1.70110	1.70090	1.70069	1.70045	1.70018	1.69988	1.69954
.060	1.78739	1.78716	1.78691	1.78662	1.78630	1.78594	1.78552
.065	1.86665	1.86638	1.86608	1.86575	1.86537	1.86494	1.86446
.070	1.93992	1.93961	1.93926	1.93887	1.93843	1.93793	1.93737
.075	2.00801	2.00765	2.00725	2.00680	2.00630	2.00572	2.00507
.080	2.07159	2.07117	2.07072	2.07021	2.06963	2.06898	2.06824
.085	2.13118	2.13072	2.13020	2.12962	2.12898	2.12824	2.12740
.090	2.18726	2.18673	2.18615	2.18551	2.18478	2.18396	2.18301
.095	2.24017	2.23959	2.23895	2.23822	2.23741	2.23650	2.23545
.100	2.29026	2.28961	2.28890	2.28810	2.28720	2.28618	2.28502
.105	2.33778	2.33706	2.33628	2.33540	2.33441	2.33328	2.33200
.110	2.38296	2.38218	2.38132	2.38035	2.37927	2.37804	2.37663
.115	2.42602	2.42517	2.42423	2.42317	2.42198	2.42064	2.41911
.120	2.46713	2.46620	2.46517	2.46403	2.46274	2.46127	2.45961
.125	2.50644	2.50543	2.50432	2.50307	2.50167	2.50009	2.49828
.130	2.54408	2.54300	2.54179	2.54045	2.53894	2.53723	2.53528
.135	2.58019	2.57902	2.57772	2.57627	2.57464	2.57280	2.57070
.140	2.61486	2.61360	2.61221	2.61065	2.60891	2.60693	2.60467
.145	2.64820	2.64685	2.64536	2.64369	2.64182	2.63970	2.63729
.150	2.68029	2.67885	2.67725	2.67547	2.67347	2.67121	2.66863

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TABLE III. - VALUES OF FUNCTION $h(X_r) = \int_{0.01}^{X_r} \frac{1}{X}(1-X^2)^{\frac{1}{k-1}} dX$ - Continued.

Velocity parameter X_r	Ratio of specific heats of gas, k						
	1.40	1.38	1.36	1.34	1.32	1.30	1.28
0.150	2.68029	2.67885	2.67725	2.67547	2.67347	2.67121	2.66863
.155	2.71120	2.70967	2.70797	2.70607	2.70393	2.70152	2.69877
.160	2.74102	2.73939	2.73758	2.73554	2.73329	2.73072	2.72780
.165	2.76980	2.76807	2.76614	2.76400	2.76159	2.75887	2.75577
.170	2.79760	2.79576	2.79373	2.79145	2.78890	2.78602	2.78273
.175	2.82448	2.82254	2.82038	2.81798	2.81528	2.81223	2.80876
.180	2.85049	2.84843	2.84616	2.84362	2.84077	2.83755	2.83388
.185	2.87566	2.87350	2.87109	2.86842	2.86541	2.86202	2.85816
.190	2.90005	2.89777	2.89524	2.89242	2.88926	2.88569	2.88163
.195	2.92368	2.92128	2.91863	2.91567	2.91235	2.90860	2.90433
.200	2.94660	2.94409	2.94130	2.93819	2.93470	2.93077	2.92629
.205	2.96884	2.96620	2.96328	2.96002	2.95637	2.95225	2.94756
.210	2.99043	2.98766	2.98460	2.98119	2.97737	2.97306	2.96815
.215	3.01139	3.00850	3.00530	3.00173	2.99774	2.99323	2.98810
.220	3.03176	3.02874	3.02539	3.02167	3.01750	3.01279	3.00744
.225	3.05155	3.04840	3.04491	3.04102	3.03667	3.03176	3.02618
.230	3.07080	3.06751	3.06387	3.05982	3.05529	3.05017	3.04436
.235	3.08952	3.08608	3.08230	3.07809	3.07336	3.06804	3.06200
.240	3.10772	3.10416	3.10022	3.09583	3.09092	3.08539	3.07911
.245	3.12544	3.12174	3.11764	3.11309	3.10799	3.10224	3.09571
.250	3.14270	3.13885	3.13459	3.12986	3.12457	3.11860	3.11183
.255	3.15949	3.15550	3.15109	3.14618	3.14069	3.13450	3.12748
.260	3.17585	3.17171	3.16714	3.16205	3.15636	3.14995	3.14268
.265	3.19178	3.18750	3.18276	3.17749	3.17160	3.16496	3.15744
.270	3.20731	3.20287	3.19796	3.19251	3.18642	3.17956	3.17178
.275	3.22244	3.21785	3.21277	3.20714	3.20084	3.19375	3.18571
.280	3.23719	3.23244	3.22720	3.22137	3.21486	3.20754	3.19925
.285	3.25156	3.24666	3.24125	3.23523	3.22851	3.22096	3.21240
.290	3.26558	3.26052	3.25493	3.24872	3.24179	3.23400	3.22518
.295	3.27925	3.27403	3.26827	3.26187	3.25472	3.24669	3.23760
.300	3.29258	3.28720	3.28126	3.27466	3.26730	3.25903	3.24967

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TABLE III - VALUES OF FUNCTION $h(X_r) = \int_{0.01}^{X_r} \frac{1}{X}(1-X^2)^{\frac{1}{K-1}} dX$ - Concluded.

Velocity parameter X_r	Ratio of specific heats of gas, k						
	1.40	1.38	1.36	1.34	1.32	1.30	1.28
0.300	3.29258	3.28720	3.28126	3.27466	3.26730	3.25903	3.24967
.305	3.30558	3.30004	3.29392	3.28713	3.27955	3.27103	3.26140
.310	3.31827	3.31256	3.30626	3.29927	3.29147	3.28271	3.27280
.315	3.33065	3.32477	3.31829	3.31110	3.30308	3.29407	3.28389
.320	3.34272	3.33668	3.33001	3.32262	3.31437	3.30512	3.29466
.325	3.35450	3.34829	3.34144	3.33384	3.32537	3.31587	3.30514
.330	3.36600	3.35962	3.35258	3.34478	3.33608	3.32633	3.31532
.335	3.37722	3.37067	3.36344	3.35543	3.34651	3.33651	3.32521
.340	3.38817	3.38144	3.37403	3.36581	3.35666	3.34641	3.33483
.345	3.39886	3.39196	3.38435	3.37593	3.36655	3.35604	3.34418
.350	3.40929	3.40221	3.39442	3.38578	3.37617	3.36541	3.35327
.355	3.41947	3.41222	3.40423	3.39538	3.38554	3.37452	3.36210
.360	3.42940	3.42198	3.41379	3.40474	3.39467	3.38339	
.365	3.43910	3.43150	3.42312	3.41385	3.40355	3.39201	
.370	3.44857	3.44078	3.43221	3.42273	3.41219		
.375	3.45781	3.44984	3.44108	3.43138	3.42061		
.380	3.46683	3.45868	3.44972	3.43981			
.385	3.47563	3.46730	3.45814	3.44802			
.390	3.48422	3.47571	3.46636				
.395	3.49261	3.48391	3.47436				
.400	3.50079	3.49192					
.405	3.50878						
.410	3.51657						

TABLE IV - VALUES OF $h(X_r)$ FOR CRITICAL VELOCITIES

k	X_{cr}	$h(X_{cr})$
1.40	0.40825	3.51384
1.38	.39957	3.49123
1.36	.39056	3.46725
1.34	.38118	3.44175
1.32	.37139	3.41453
1.30	.36115	3.38537
1.28	.35044	3.35404

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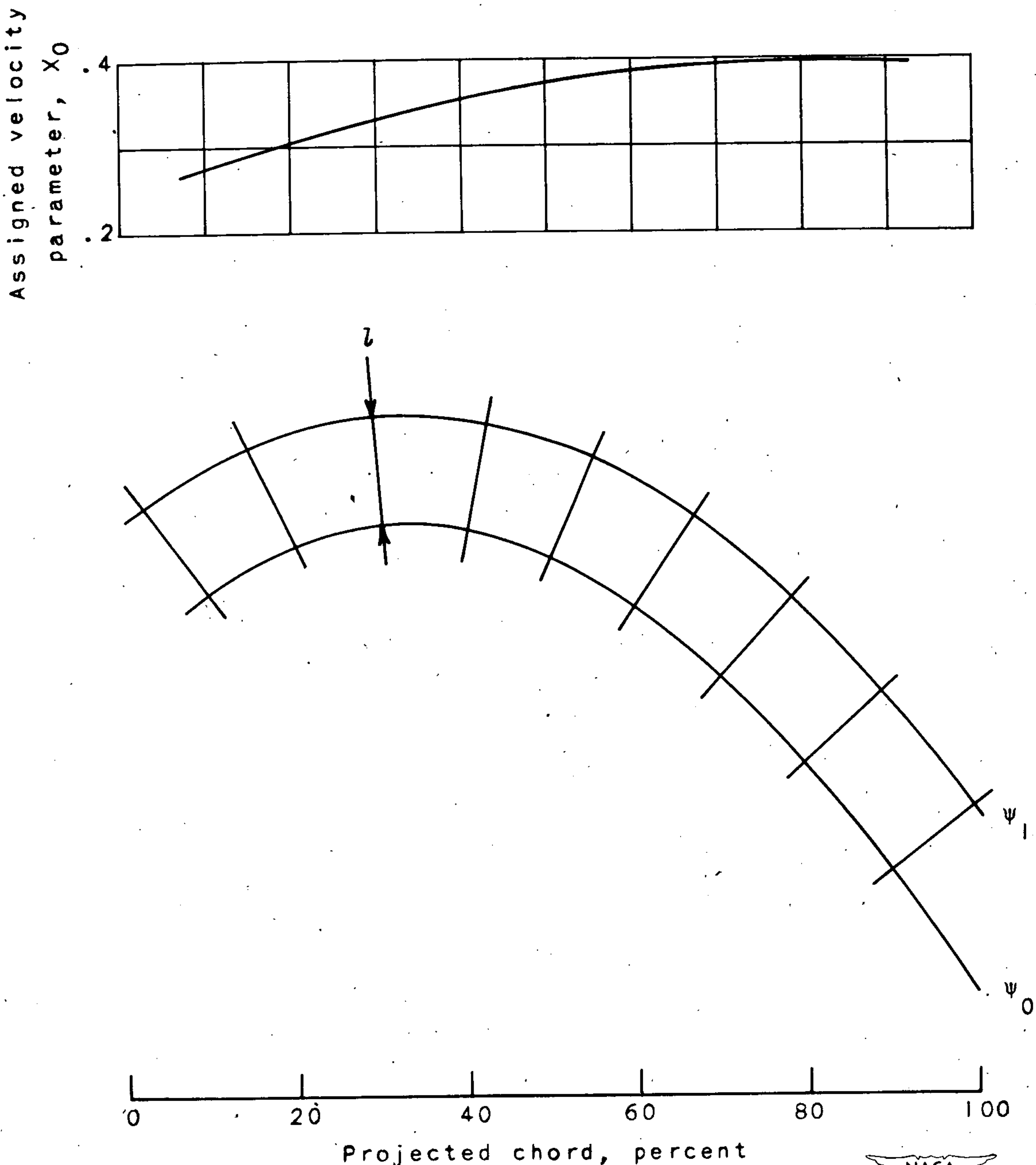

Figure 1. - Typical assigned suction-surface contour, typical assigned velocity distribution, and construction of first stream filament.

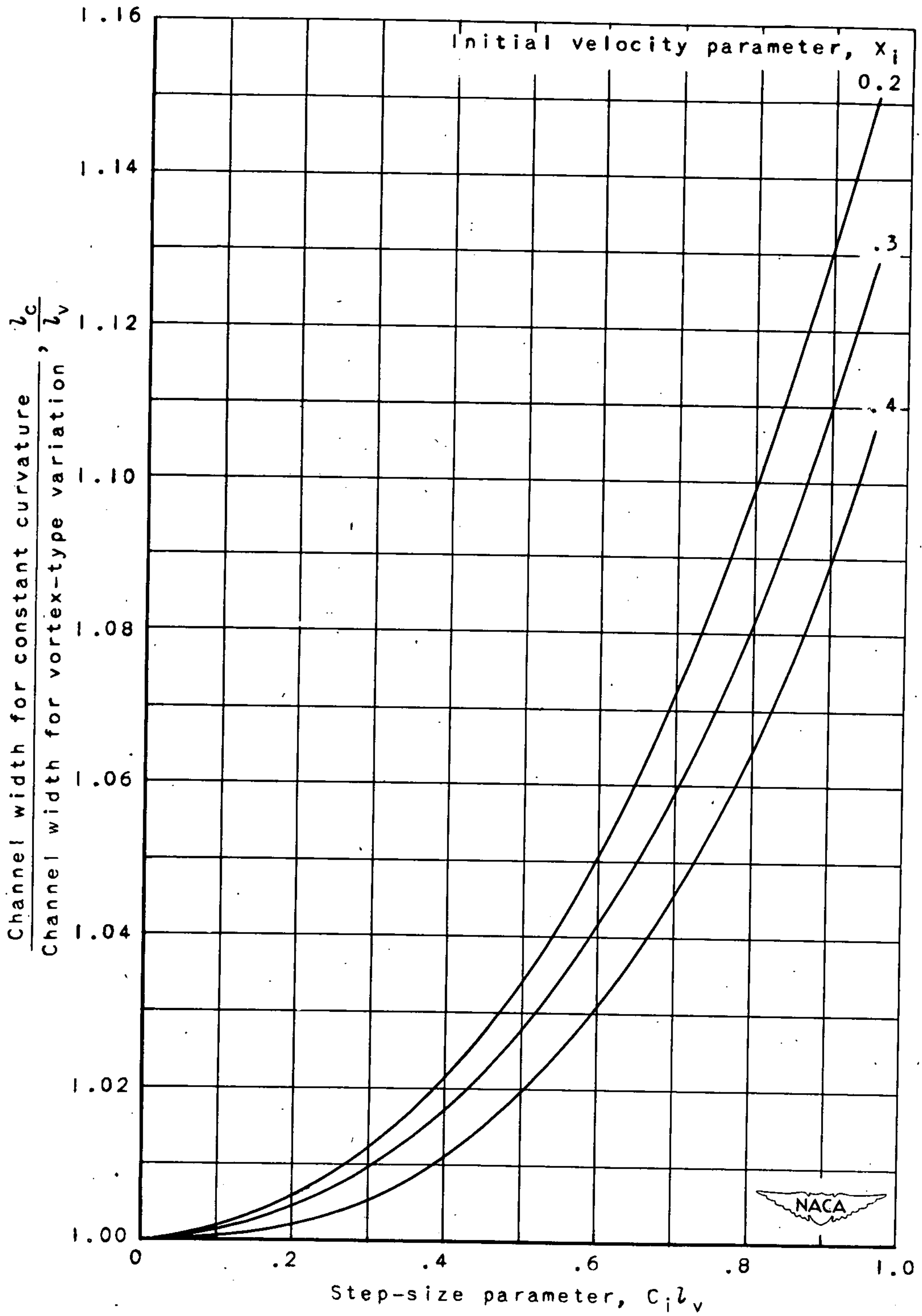

Figure 2. - Effect of step-size parameter on ratio of channel width for constant curvature to channel width for vortex-type variation. Ratio of specific heats k, 1.4.

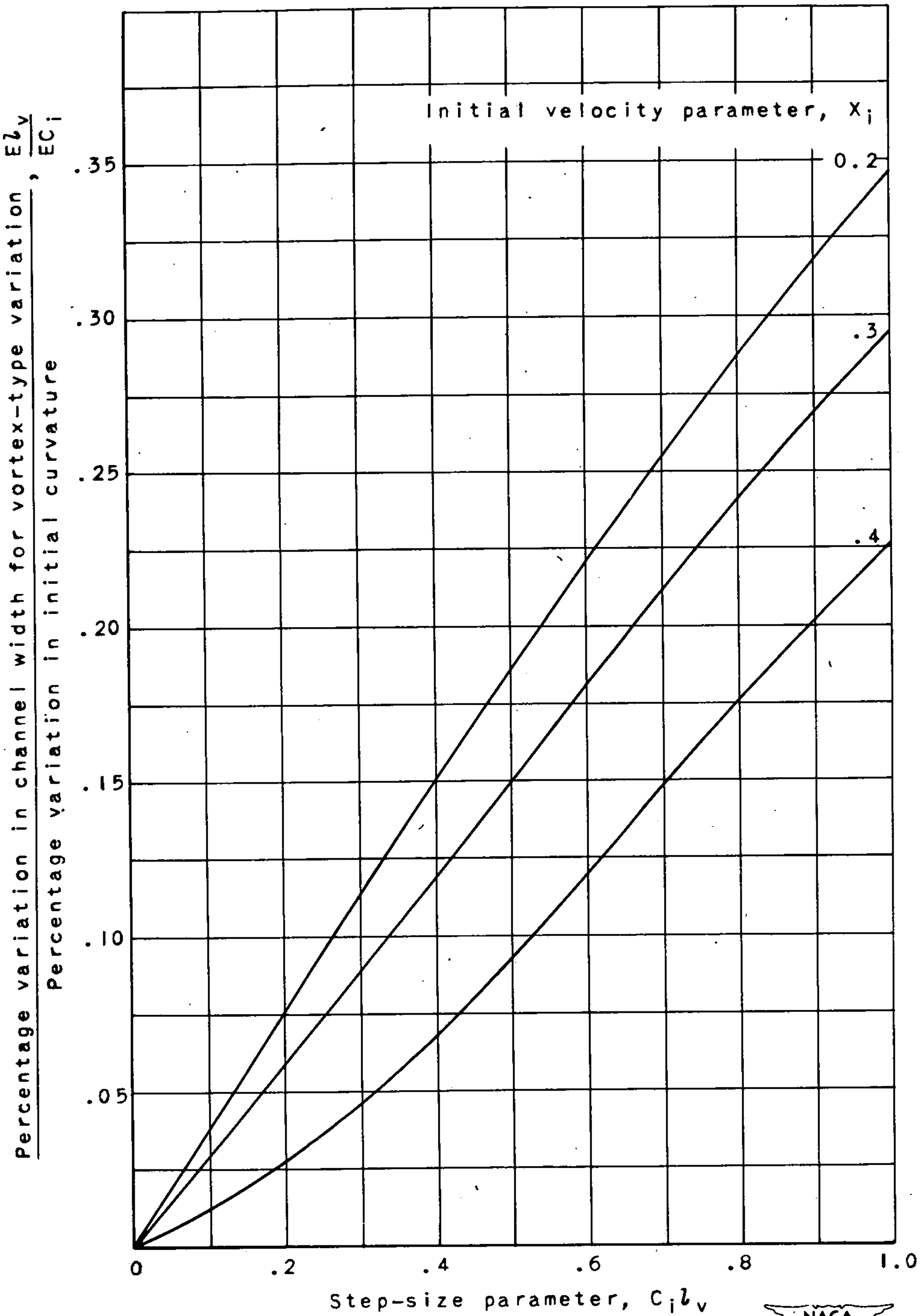

Figure 3. - Effect of step-size parameter on ratio of percentage variation in channel width for vortex-type variation to percentage variation in initial curvature from equation (15). Ratio of specific heats k, 1.4.

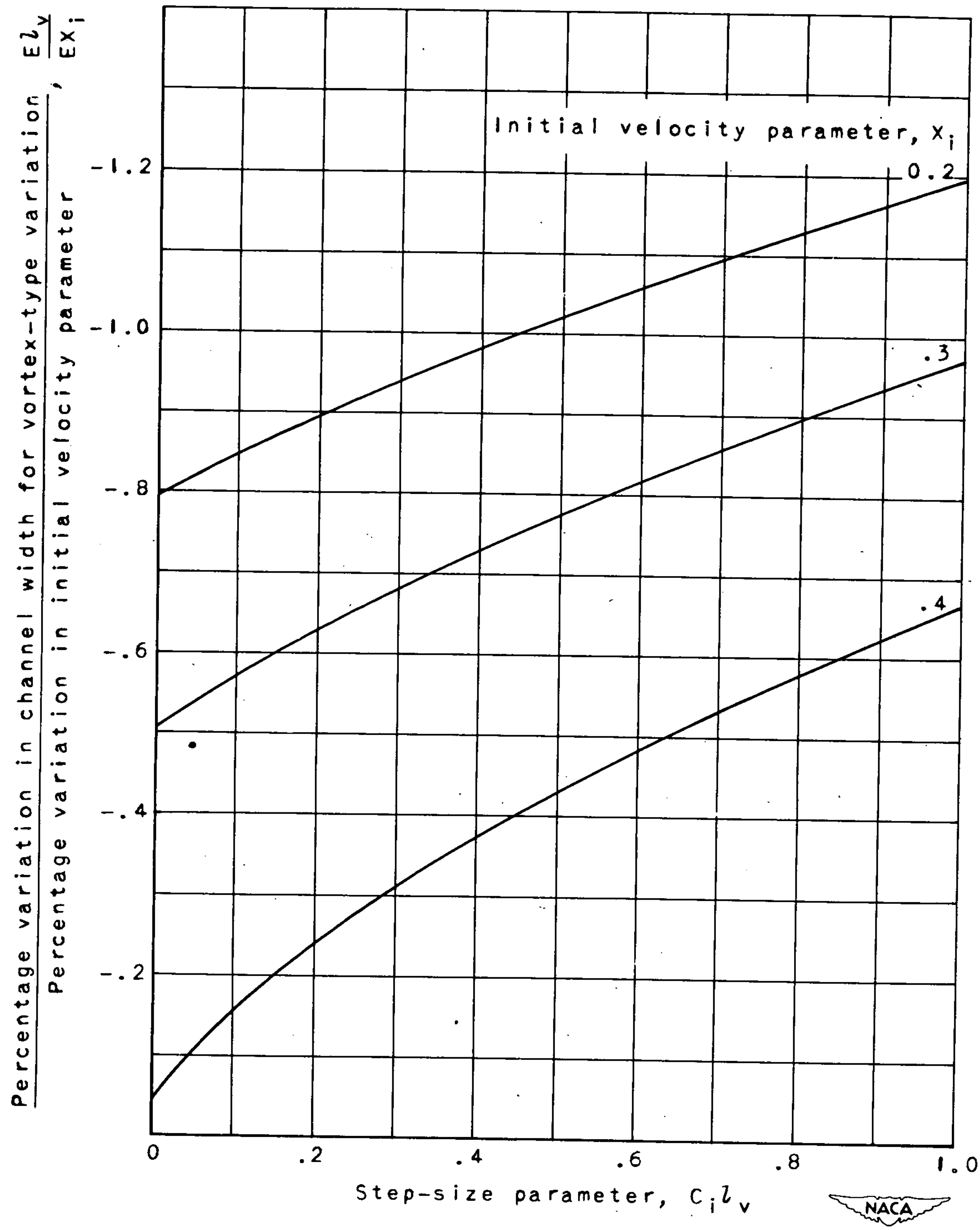

Figure 4. - Effect of step-size parameter on ratio of percentage variation in channel width for vortex-type variation to percentage variation in initial velocity parameter from equation (16). Ratio of specific heats k, 1.4.

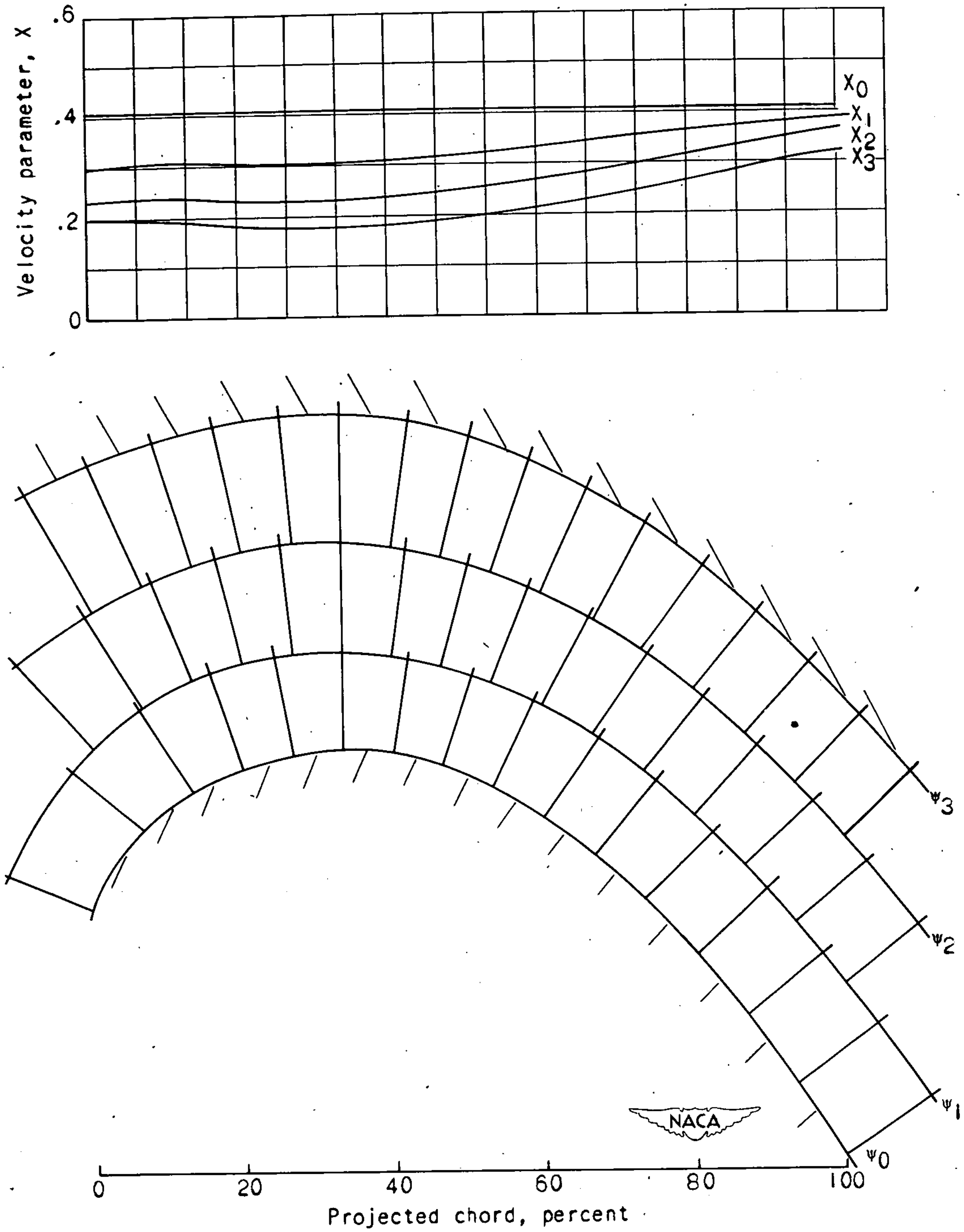

Figure 5. - Assigned suction-surface contour, assigned velocity distribution, and channel construction for illustrative example.

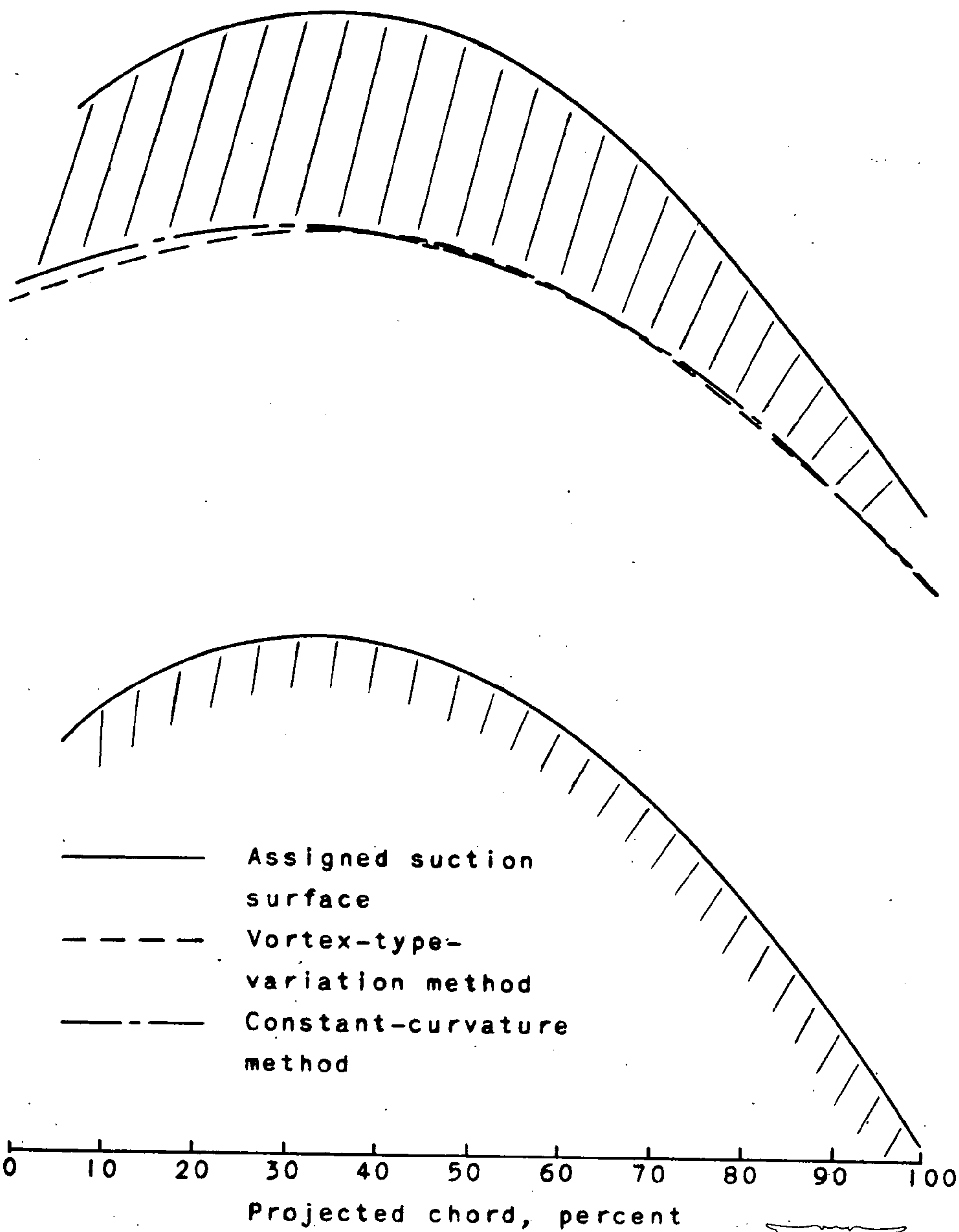

Figure 6. - Comparison of channels constructed by constant-curvature and vortex-type-variation methods for illustrative example.